Phys 410
Spring 2013
Lecture #15 Summary
25 February, 2013

The Coriolis force $\vec{F}_{Cor} = 2m\dot{\vec{r}} \times \vec{\Omega}$ depends on the state of motion of the object. In fact it resembles the force on a charged particle in a magnetic field. The 'charge' is $2m$ and the 'magnetic field' is the angular velocity vector $\vec{\Omega}$. The particle will be deflected as it travels through this 'field'. In the northern hemisphere the deflection is to the right, while in the southern hemisphere it is in the opposite direction because $\vec{\Omega}$ has a substantial component into the ground (hence the phrase 'down under'). The magnitude of the Coriolis force for an object on the surface of the earth moving at 50 m/s is quite small, resulting in an acceleration of at most 0.007 m/s^2. We did a demonstration showing the deflection of a stream of water under the influence of the Coriolis force.

We next considered the motion of the Foucault pendulum. The demonstration showed that the pendulum moves in a fixed plane, as seen from an inertial reference frame. However, in a rotating reference frame, the pendulum appears to move in a series of planes that rotate clockwise, as seen from above. The pendulum is made of a light wire of length L supporting a bob of mass m. The equation of motion of the bob as seen in the non-inertial frame is $m\ddot{\vec{r}} = \vec{F}_{net} + 2m\dot{\vec{r}} \times \vec{\Omega} + m(\vec{\Omega} \times \vec{r}) \times \vec{\Omega}$, where the net force identified from an inertial reference frame is the vector sum of tension in the wire and gravity: $\vec{F}_{net} = \vec{T} + m\vec{g}_0$. This is the bare gravity force that points toward the center of the earth. Last time we saw that bare gravity can be combined with the centrifugal force and re-named effective gravity: $\vec{g} = \vec{g}_0 + \Omega^2 R \sin\theta\, \hat{\rho}$. We designate "up" or the +z-direction to be the direction away from $\vec{g}$, and y to be the "north" direction, and x to be the "east" direction. In this way, the angular velocity vector for the earth $\vec{\Omega}$ points somewhere in the y-z plane.

The z-motion of the bob is fairly simple, essentially reducing to the statement that $T \cong mg$. The tension in the horizontal xy-plane is $T_x = -mgx/L$, and $T_y = -mgy/L$. The Coriolis force is found from the cross product $2m\dot{\vec{r}} \times \vec{\Omega}$. We write $\dot{\vec{r}} = (\dot{x}, \dot{y}, \dot{z})$ and $\vec{\Omega} = (0, \Omega\sin\theta, \Omega\cos\theta)$. After carrying out the cross product and putting the results into the equation of motion, broken down into components, we get: $m\ddot{x} = -\frac{mgx}{L} + 0 + 2m(\dot{y}\Omega\cos\theta - \dot{z}\Omega\sin\theta)$, and $m\ddot{y} = -\frac{mgy}{L} + 0 - 2m\dot{x}\Omega\cos\theta$. We shall drop the $\dot{z}$ term in the x-equation because it is the product of two small velocities, define the constants $\omega_0^2 \equiv g/L$, and $\Omega_z \equiv \Omega\cos\theta$, to get two coupled equations of motion:

$$\ddot{x} - 2\dot{y}\Omega_z + \omega_0^2 x = 0$$

$$\ddot{y} + 2\dot{x}\Omega_z + \omega_0^2 y = 0$$

The first and third terms alone would give un-coupled simple harmonic motion in the xy-plane. The coupling terms look like a form of dissipation (of the form $F_{dis} = -bv$) but in fact they represent a coupling of energy from one direction of motion to the other. The energy in the oscillations sloshes back and forth between x and y.